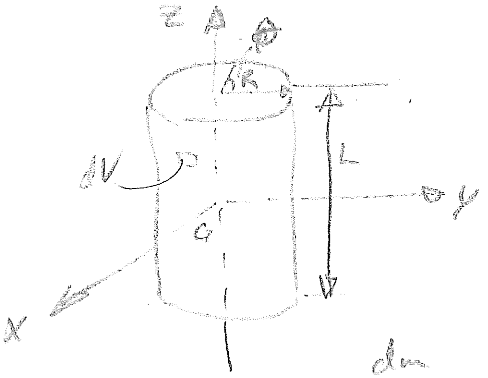
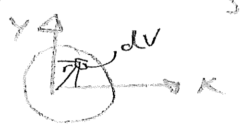
1

Ex Homogen solid cylinder



Vid rot kring z



$$r^2 = x^2 + y^2$$
$$= (r\cos\theta)^2 + (r\sin\theta)^2$$
$$= r^2(\cos^2\theta + \sin^2\theta) = r^2$$

$$I_{zz} = \int_{Vol} r^2 \overbrace{\rho\, dV}^{dm} = \rho \int_{Vol} r^2\, dV$$

(se även s. 668 för variant på framställning)

Hur väljer vi volymselementet dV?

Om vi väljer att beskriva den cirulära tvärsnittsarean med polära koordinater r o θ kan vi skriva trippelintegralen med följande gränser

$$I_{zz} = \rho \int_{z=-\frac{L}{2}}^{L/2} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} r^2 (r\,dr\,d\theta\,dz)$$

$$= \rho \int_{z=-\frac{L}{2}}^{L/2} \int_{\theta=0}^{2\pi} \left[\frac{r^4}{4}\right]_0^R d\theta\, dz$$

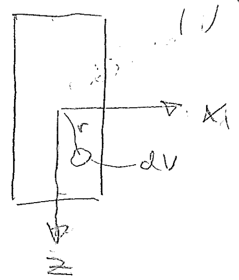
$$= \rho \frac{R^4}{4} \int_{z=-L/2}^{L/2} \left[\theta\right]_0^{2\pi} dz$$

$$= \rho \pi \frac{R^4}{2} \left[z\right]_{-L/2}^{L/2}$$

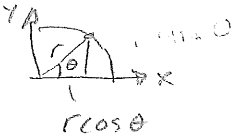
$$I_{zz} = \rho \pi \frac{R^4}{2} L = \underbrace{\rho \underbrace{\pi R^2 L}_{\text{Volym}}}_{\text{massan}} \frac{R^2}{2} = \frac{mR^2}{2}$$ (oberoende av L!)

forts. →

Vid rotation kring y-axeln



$r^2 = x^2 + z^2$



$$I_{yy} = \rho \int_{Vol} (x^2 + z^2)\, dV$$

$$I_{yy} = \rho \int_{z=-L/2}^{L/2} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} (x^2 + z^2)(r\, dr\, d\theta\, dz)$$

$$= \rho \int_{z=-L/2}^{L/2} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} ((r\cos\theta)^2 + z^2)\, r\, dr\, d\theta\, dz$$

$$= \rho \int_{z=-\frac{L}{2}}^{L/2} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} (r^3\cos^2\theta + z^2 r)\, dr\, d\theta\, dz$$

$$= \text{——} \left[\frac{r^4}{4}\cos^2\theta + z^2\frac{r^2}{2}\right]_0^R d\theta\, dz$$

$$= \text{——} \int_{\theta=0}^{2\pi} \left(\frac{R^4}{4}\cos^2\theta + z^2\frac{R^2}{2}\right) d\theta\, dz$$

$$= \rho \int_{z=-\frac{L}{2}}^{L/2} \left[\frac{R^4}{4}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + z^2\frac{R^2}{2}\theta\right]_0^{2\pi} dz$$

$$= \rho \int_{z=-\frac{L}{2}}^{L/2} \left(\frac{R^4}{4}\left(\frac{2\pi}{2} + \frac{\overset{=0}{\sin 4\pi}}{4}\right) + z^2\frac{R^2}{2}2\pi\right) dz$$

$$= \rho \int_{z=-\frac{L}{2}}^{L/2} \left(\frac{\pi R^4}{4} + \pi R^2 z^2\right) dz$$

$$= \rho \left[\frac{\pi R^4}{4}z + \frac{\pi R^2}{3}z^3\right]_{-L/2}^{L/2} = \underbrace{\rho \underbrace{\pi R^2 L}_{\text{Volymen}}}_{\text{massan}} \left(\frac{R^2}{4} + \frac{L^2}{12}\right)$$

$$I_{yy} = \frac{mR^2}{4} + \frac{mL^2}{12}$$

I_{yy} för tunn stång!

I_{yy} för tunn skiva!