LÖSNINGAR

TILL

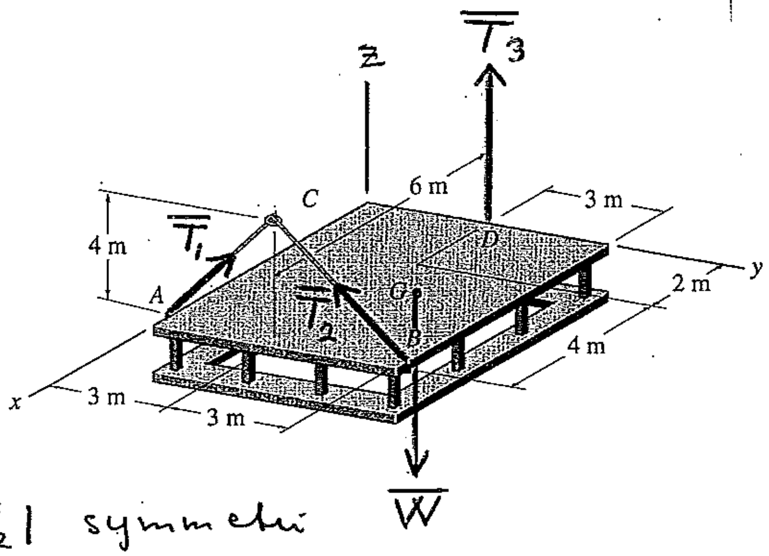
TENTAMEN I

MEKANIK TMMI39 f.k.

090416

1)

090416



$|\overline{T}_1| = |\overline{T}_2|$ symmetri

$\Sigma \overline{M}_A = \overline{0}$

$\overline{r}_{AB} \times \overline{T}_2 + \overline{r}_{AD} \times \overline{T}_3 + \overline{r}_{AG} \times \overline{W} = \overline{0}$

$\overline{r}_{AB} = 6\overline{j} \quad \overline{r}_{AD} = -6\overline{i} + 3\overline{j} \quad \overline{r}_{AG} = -4\overline{i} + 3\overline{j}$

$\overline{T}_2 = T_2 \frac{\overline{BC}}{|\overline{BC}|} = T_2 \frac{-3\overline{j} + 4\overline{k}}{\sqrt{3^2 + 4^2}} = -0{,}6\,T_2\,\overline{j} + 0{,}8\,T_2\,\overline{k}$

$\overline{T}_3 = T_3\,\overline{k} \qquad \overline{W} = -3g\,\overline{k}$ kN

$$\Sigma \overline{M}_A = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 6 & 0 \\ 0 & -0{,}6T_2 & 0{,}8T_2 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -6 & 3 & 0 \\ 0 & 0 & T_3 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -4 & 3 & 0 \\ 0 & 0 & -3g \end{vmatrix} = \overline{0}$$

$$4{,}8T_2\,\overline{i} + \left(3T_3\,\overline{i} + 6T_3\,\overline{j}\right) + \left((-9g)\,\overline{i} - 12g\,\overline{j}\right) = \overline{0}$$

x-led ① $4{,}8\,T_2 + 3T_3 - 9g = 0$

y-led ② $6T_3 - 12g = 0$

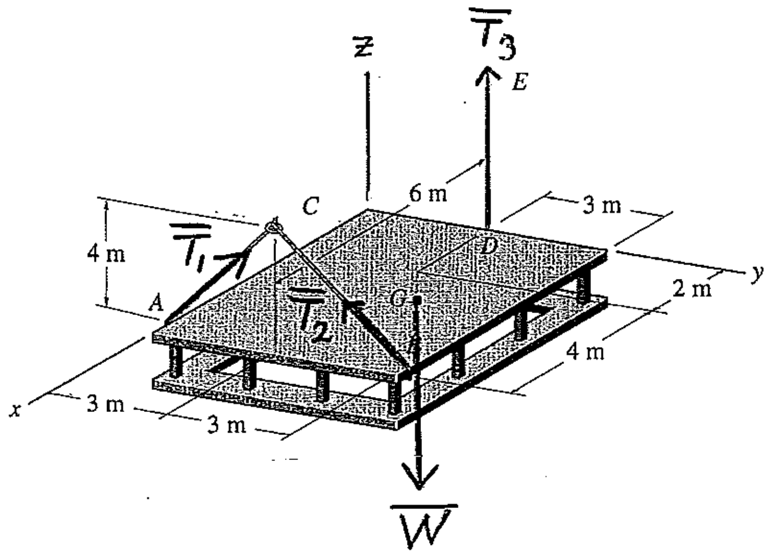
② $T_3 = 2g = 19{,}62$ kN

① $T_2 = 6{,}131$ kN

$T_1 = T_2 = 6{,}131$ kN

Svar: $T_1 = T_2 = 6{,}13$ kN. $T_3 = 19{,}6$ kN.

1)



$$\Sigma \overline{F} = \overline{0}$$

$$\overline{T_1} + \overline{T_2} + \overline{T_3} + \overline{W} = \overline{0}$$

$$\overline{T_1} = T_1 \frac{\overline{AC}}{|\overline{AC}|} = T_1 \frac{3\overline{j} + 4\overline{k}}{\sqrt{3^2 + 4^2}} = T_1 (0{,}6\overline{j} + 0{,}8\overline{k})$$

$$\overline{T_2} = T_2 \frac{\overline{BC}}{|\overline{BC}|} = T_2 \frac{-3\overline{j} + 4\overline{k}}{\sqrt{3^2 + 4^2}} = T_2 (-0{,}6\overline{j} + 0{,}8\overline{k})$$

$$\overline{T_3} = T_3 \overline{k} \qquad \overline{W} = -3g\overline{k} \text{ kN}$$

$$T_1 (0{,}6\overline{j} + 0{,}8\overline{k}) + T_2 (-0{,}6\overline{j} + 0{,}8\overline{k}) +$$
$$+ T_3 \overline{k} - 3g\overline{k} = 0$$

y-led ① $0{,}6 T_1 - 0{,}6 T_2 = 0$

z-led ② $0{,}8 T_1 + 0{,}8 T_2 + T_3 - 3g = 0$

① $T_1 = T_2$

② $T_3 = 3g - 1{,}6 T_1$

$$\Sigma \overline{M_A} = \overline{0}$$

$$\overline{r_{AB}} \times \overline{T_2} + \overline{r_{AD}} \times \overline{T_3} + \overline{r_{AG}} \times \overline{W} = \overline{0}$$

$$\overline{r_{AB}} = 6\overline{j} \qquad \overline{r_{AD}} = -6\overline{i} + 3\overline{j} \qquad \overline{r_{AG}} = -4\overline{i} + 3\overline{j}$$

forts.

1) forts.

$$\Sigma \overline{M}_A = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 6 & 0 \\ 0 & -0{,}6T_2 & 0{,}8T_2 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -6 & 3 & 0 \\ 0 & 0 & T_3 \end{vmatrix} \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -4 & 3 & 0 \\ 0 & 0 & -3g \end{vmatrix} = \overline{0}$$

$$4{,}8\, T_2\, \overline{i} + \left(3\, T_3\, \overline{i} + 6\, T_3\, \overline{j}\right) + \left((-9g)\,\overline{i} - 12g\,\overline{j}\right) = \overline{0}$$

x-led (3) $\quad 4{,}8\, T_2 + 3T_3 - 9g = 0$

y-led (4) $\quad 6\, T_3 - 12g = 0$

(4) $\quad T_3 = 2g = 19{,}62 \text{ kN}$

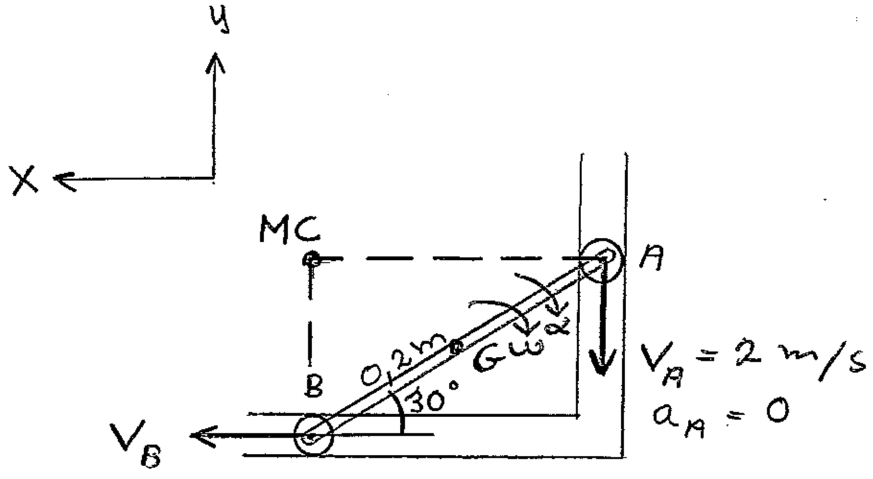
(3) $\quad T_2 = (9g - 6g)/4{,}8$

$\quad T_2 = 6{,}131 \text{ kN}$

(1) $\quad T_1 = T_2 = 6{,}131 \text{ kN}$

Svar: $T_1 = T_2 = 6{,}13$ kN. $T_3 = 19{,}6$ kN.

2)



$$\bar{a}_G = \bar{a}_A + \bar{\alpha} \times \overline{AG} + \bar{\omega} \times (\bar{\omega} \times \overline{AG})$$

Bestäm $\bar{\omega}$ och $\bar{\alpha}$

$$V_A = \omega r \Rightarrow 2 = \omega \cdot 0{,}2 \cos 30^\circ \qquad \underline{\omega = 11{,}547 \text{ rad/s}}$$

$$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \overline{AB} + \bar{\omega} \times (\bar{\omega} \times \overline{AB})$$

$$\begin{Bmatrix} a_B \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \alpha \end{Bmatrix} \times \begin{Bmatrix} 0{,}2\cos 30^\circ \\ -0{,}2 \sin 30^\circ \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \left[\begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \begin{Bmatrix} 0{,}2\cos 30^\circ \\ -0{,}2 \sin 30^\circ \\ 0 \end{Bmatrix} \right]$$

$$\begin{Bmatrix} a_B \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0{,}2 \sin 30^\circ \cdot \alpha \\ 0{,}2 \cos 30^\circ \cdot \alpha \\ 0 \end{Bmatrix} + \begin{Bmatrix} -0{,}2 \cos 30^\circ \cdot \omega^2 \\ 0{,}2 \sin 30^\circ \cdot \omega^2 \\ 0 \end{Bmatrix}$$

y-led $\quad 0 = 0{,}2 \cos 30^\circ \cdot \alpha + 0{,}2 \sin 30^\circ \cdot \omega^2$

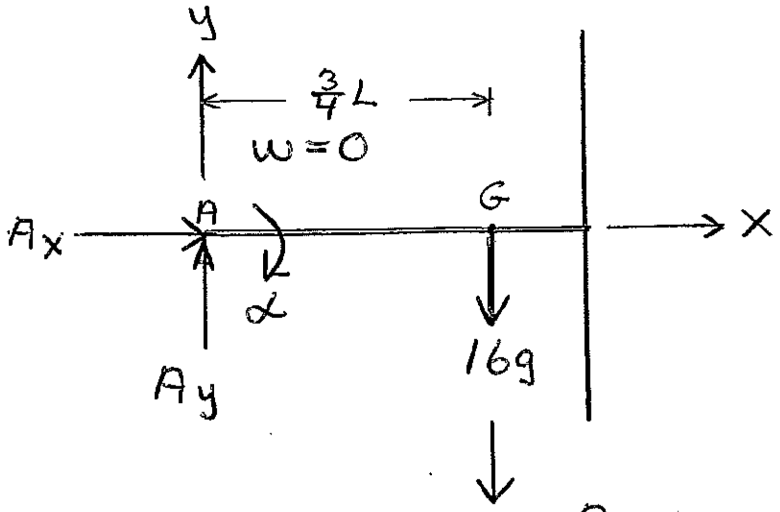
$$\underline{\alpha = -76{,}98 \text{ rad/s}^2}$$

$$\bar{a}_G = \begin{Bmatrix} 0 \\ 0 \\ \alpha \end{Bmatrix} \times \begin{Bmatrix} 0{,}1\cos 30^\circ \\ -0{,}1 \sin 30^\circ \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \left[\begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \begin{Bmatrix} 0{,}1\cos 30^\circ \\ -0{,}1 \sin 30^\circ \\ 0 \end{Bmatrix} \right]$$

$$\bar{a}_G = \begin{Bmatrix} 0{,}1 \sin 30^\circ \cdot \alpha \\ 0{,}1 \cos 30^\circ \cdot \alpha \\ 0 \end{Bmatrix} + \begin{Bmatrix} -0{,}1 \cos 30^\circ \cdot \omega^2 \\ 0{,}1 \sin 30^\circ \cdot \omega^2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -15{,}40 \\ 0 \\ 0 \end{Bmatrix}$$

Svar: $\underline{\underline{\bar{a}_G = -15{,}4 \,\bar{i} \text{ m/s}^2}}$

3)



$\Sigma F_x = m a_{Gx}$ ① $A_x = 0 \quad (a_{Gx} = 0)$

$\Sigma F_y = m a_{Gy}$ ② $A_y - 16g = 16 \cdot (-1{,}5\alpha)$

$\Sigma M_G = I_G \alpha$ ↻ ③ $A_y \cdot \frac{3}{4} \cdot 2 = I_G \cdot \alpha$

$I_G = \left[\frac{1}{12} m_1 L^2 + m_1 \left(\frac{L}{4}\right)^2\right] \cdot 2 \quad (m_1 = 8\,kg)$

$I_G = \frac{28}{3}\ kg\,m^2$

③ $A_y \cdot \frac{3}{4} \cdot 2 = \frac{28}{3}\alpha$

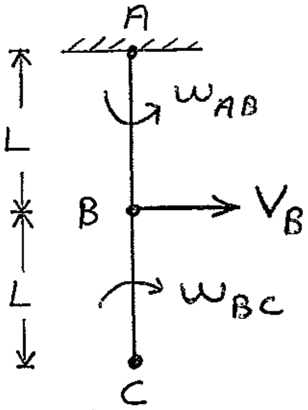
$\alpha = \frac{9}{56} A_y$

② $A_y - 16g = 16 \cdot \left(-1{,}5 \cdot \frac{9}{56} A_y\right)$

$A_y = 32{,}32\ N$

Svar: $A_x = 0 \quad A_y = 32{,}3\ N$

4)

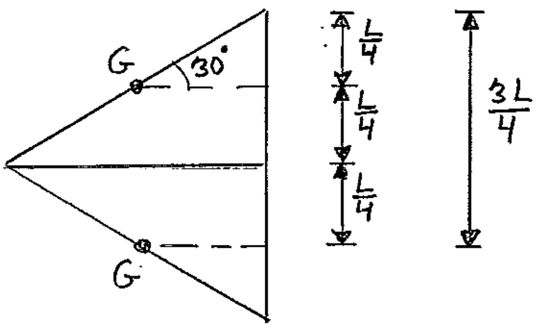


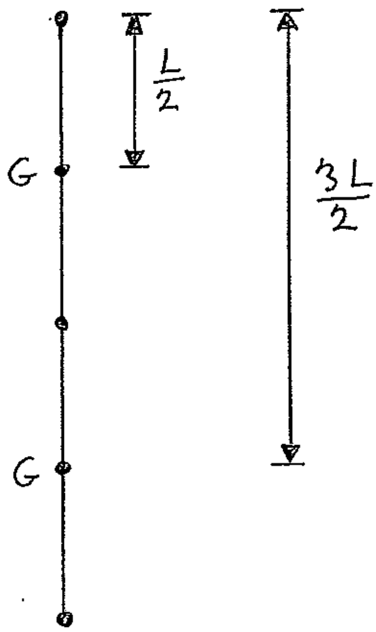
I det raka läget, dvs då $\theta = 90^\circ$ är C momentancentrum

$V_B = L\omega_{AB} = L\omega_{BC}$

$\omega_{AB} = \omega_{BC} = \omega$

Energiekvationen $U = \Delta T + \Delta V_g + \Delta V_e$





$U = \Delta V_e = 0$

$\Delta T = T_2 - T_1 = T_2 - 0$

$T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2 \qquad V_G = \frac{L}{2} \cdot \omega$

$\Delta T = \left(\frac{1}{2} m \cdot \left(\frac{L}{2}\omega\right)^2 + \frac{1}{2} \cdot \frac{1}{12} m L^2 \cdot \omega^2\right) \cdot 2 = \frac{1}{3} m L^2 \omega^2$

$\Delta V_g = -mg\left(\frac{L}{2} - \frac{L}{4}\right) - mg\left(\frac{3L}{2} - \frac{3L}{4}\right) = -mgL$

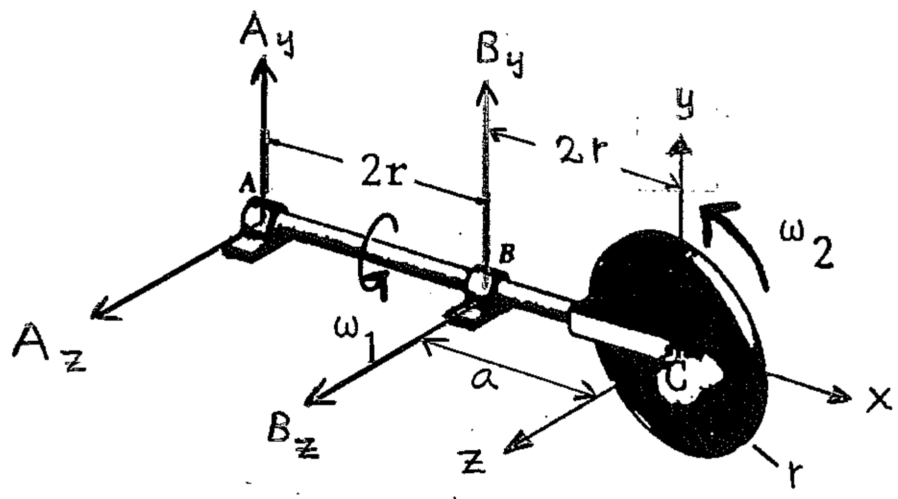
$0 = \frac{1}{3} \cancel{m} L^2 \omega^2 - \cancel{m} g L$

$\omega = \sqrt{\frac{3g}{L}}$

Svar: $\omega_{AB} = \omega_{BC} = \sqrt{\frac{3g}{L}}$

5)

09 04 16



Endast krafter som uppstår p.g.a. rotationen söks. Tyngdkraften tas då ej med.

ω_1 vrider xyz

$$\overline{\omega} = \overline{\omega}_1 + \overline{\omega}_2 = \begin{Bmatrix} \omega_1 \\ 0 \\ \omega_2 \end{Bmatrix}$$

$$\overline{M}_C = \overline{M}_G = \left(\frac{dH_G}{dt}\right)_{xyz} + \overline{\omega}_1 \times \overline{H}_G$$

$$\overline{H}_G = \begin{bmatrix} I_{xx} & - & 0 \\ 0 & - & 0 \\ 0 & - & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_1 \\ 0 \\ \omega_2 \end{bmatrix} = \begin{Bmatrix} I_{xx}\omega_1 \\ 0 \\ I_{zz}\omega_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ A_z(2r+2r) + B_z \cdot 2r \\ -A_y(2r+2r) - B_y \cdot 2r \end{Bmatrix} = \overline{0} + \begin{Bmatrix} \omega_1 \\ 0 \\ 0 \end{Bmatrix} \times \begin{Bmatrix} I_{xx}\omega_1 \\ 0 \\ I_{zz}\omega_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -I_{zz}\omega_1\omega_2 \\ 0 \end{Bmatrix}$$

① $A_z \cdot 4r + B_z \cdot 2r = -I_{zz}\omega_1\omega_2$

② $-A_y \cdot 4r - B_y \cdot 2r = 0$

$\Sigma \overline{F} = m\overline{a}_G = \overline{0}$ G ligger stilla

③ $A_y + B_y = 0 \Rightarrow A_y = -B_y$

④ $A_z + B_z = 0 \Rightarrow A_z = -B_z$

① $A_z = -I_{zz}\omega_1\omega_2/2r = \frac{1}{2}mr^2\omega_1\omega_2/2r = -\frac{mr}{4}\omega_1\omega_2$

② $B_y = 0$

Svar: $A_y = B_y = 0 \quad A_z = -\frac{mr}{4}\omega_1\omega_2 \quad B_z = \frac{mr}{4}\omega_1\omega_2$