LÖSNINGAR

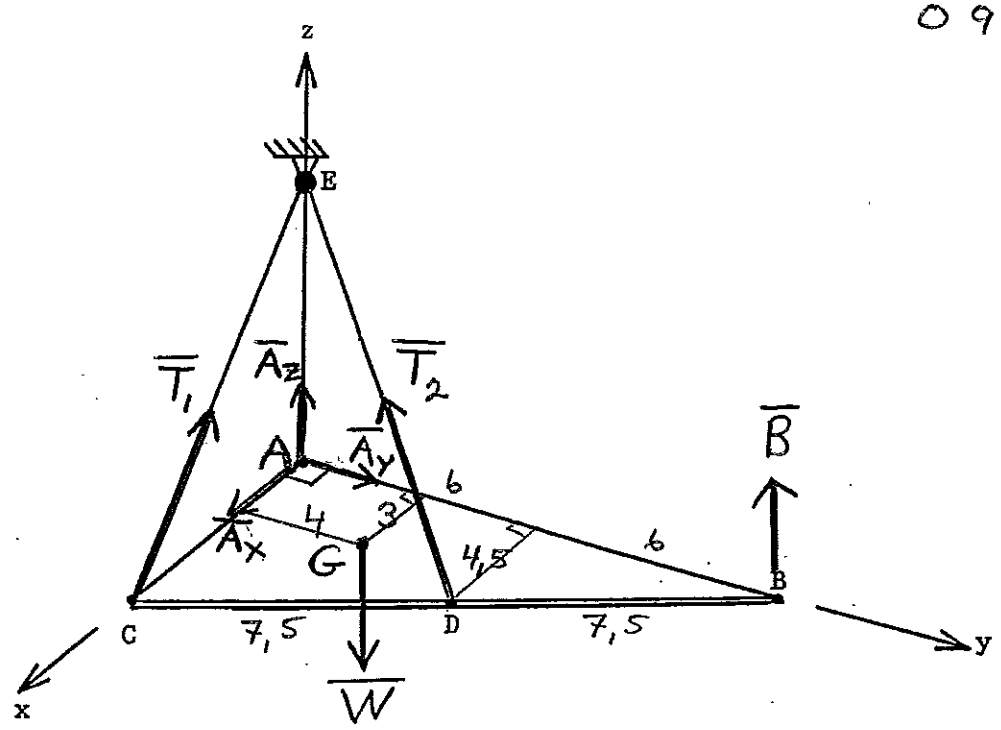
TILL

TENTAMEN I

MEKANIK TMMI39 f.k.

090822

1/



$$\Sigma \overline{M}_A = \overline{0}$$

$$\Sigma \overline{M}_A = \overline{r}_{AC} \times \overline{T}_1 + \overline{r}_{AD} \times \overline{T}_2 + \overline{r}_{AB} \times \overline{B} + \overline{r}_{AG} \times \overline{W}$$

$$\overline{r}_{AC} = 9\overline{i} \quad \overline{r}_{AD} = 4{,}5\overline{i} + 6\overline{j} \quad \overline{r}_{AB} = 12\overline{j} \quad \overline{r}_{AG} = 3\overline{i} + 4\overline{j}$$

$$\overline{T}_1 = T_1 \frac{\overline{CE}}{|CE|} = T_1 \frac{-9\overline{i} + 12\overline{k}}{\sqrt{9^2 + 12^2}} = T_1 (-0{,}6\overline{i} + 0{,}8\overline{k})$$

$$\overline{T}_2 = T_2 \frac{\overline{DE}}{|DE|} = T_2 \frac{-4{,}5\overline{i} - 6\overline{j} + 12\overline{k}}{\sqrt{4{,}5^2 + 6^2 + 12^2}} = T_2 (-0{,}318\overline{i} - 0{,}424\overline{j} + 0{,}848\overline{k})$$

$$\overline{B} = 200g\overline{k} \quad \overline{W} = -800g\overline{k}$$

$$\Sigma \overline{M}_A = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 9 & 0 & 0 \\ -0{,}6T_1 & 0 & 0{,}8T_1 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 4{,}5 & 6 & 0 \\ -0{,}318T_2 & -0{,}424T_2 & 0{,}848T_2 \end{vmatrix} +$$

$$+ \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 12 & 0 \\ 0 & 0 & 200g \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 3 & 4 & 0 \\ 0 & 0 & -800g \end{vmatrix} = \overline{0}$$

$$\begin{Bmatrix} 0 \\ -7{,}2T_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 5{,}088T_2 \\ -3{,}816T_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 2400g \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -3200g \\ 2400g \\ 0 \end{Bmatrix} = \overline{0}$$

forts.

1 forts

x-led ① $5,088\,T_2 + 2400\,g - 3200\,g = 0$

y-led ② $-7,2\,T_1 - 3,816\,T_2 + 2400\,g = 0$

① $T_2 = 157,2\,g$ N

② $T_1 = 250,0\,g$ N

Svar: $T_1 = 250\,g$ N och $T_2 = 157\,g$ N

2)

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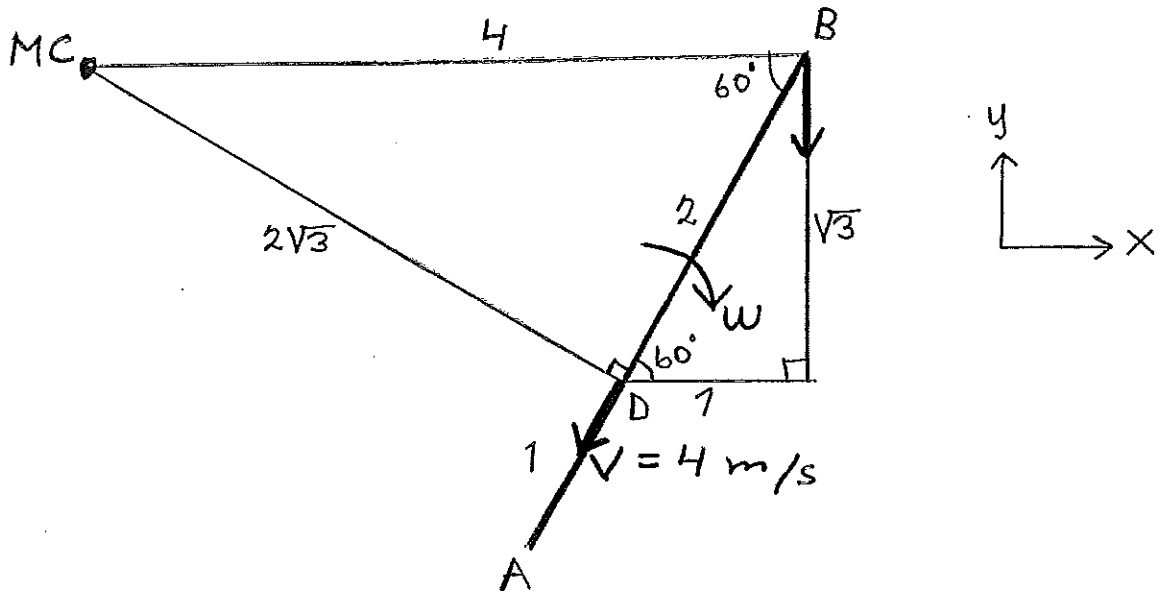

$$V = w \cdot 2\sqrt{3} \Rightarrow w = \frac{V}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} \text{ rad/s}$$

$$\overline{V_A} = \overline{V_D} + \overline{V_{A/D}} = \overline{V_D} + \overline{w} \times \overline{DA}$$

$$\overline{V_A} = \begin{Bmatrix} -4\cos 60^\circ \\ -4\sin 60^\circ \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -\frac{2}{\sqrt{3}} \end{Bmatrix} \times \begin{Bmatrix} -1\cos 60^\circ \\ -1\sin 60^\circ \\ 0 \end{Bmatrix}$$

$$\overline{V_A} = \begin{Bmatrix} -2 \\ -2\sqrt{3} \\ 0 \end{Bmatrix} + \begin{Bmatrix} -1 \\ \frac{1}{\sqrt{3}} \\ 0 \end{Bmatrix} = \begin{Bmatrix} -3 \\ -\frac{5}{\sqrt{3}} \\ 0 \end{Bmatrix}$$

Svar: $\overline{V_A} = -3\,\bar{i} - \frac{5}{\sqrt{3}}\,\bar{j}$ m/s

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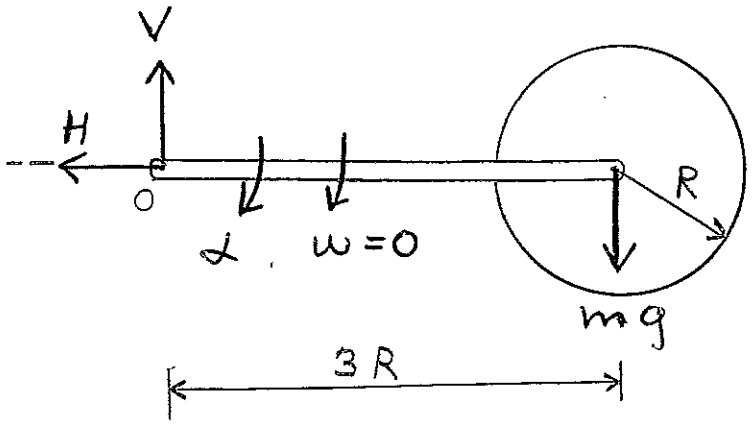
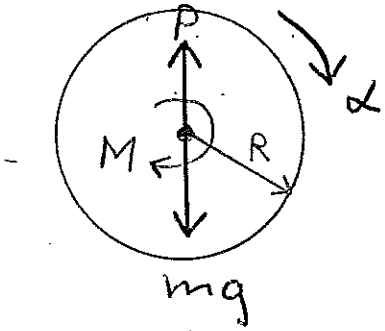
3)





$\Sigma F_n = m a_n = m r \omega^2 = 0 \Rightarrow$

$\Rightarrow H = 0$

$\Sigma M_0 = I_0 \cdot \alpha$

$mg \cdot 3R = I_0 \cdot \alpha$

$I_0 = I_G + m d^2 = \frac{1}{2} m R^2 + m(3R)^2 = \frac{19}{2} m R^2$

$mg \cdot 3R = \frac{19}{2} m R^2 \cdot \alpha$

$\alpha = \frac{6g}{19R}$

Skiran

$\Sigma F_y = m a_y$

$mg - P = ma$ $\qquad a = 3R\alpha = 3R \cdot \frac{6g}{19R} = \frac{18}{19} g$

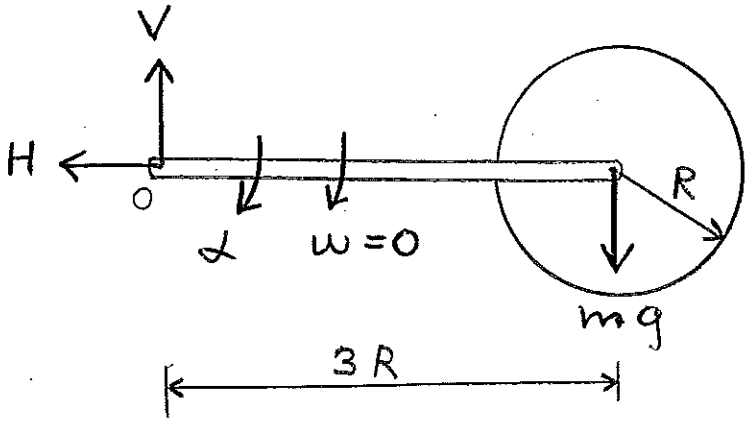
$mg - P = m \cdot \frac{18}{19} g$

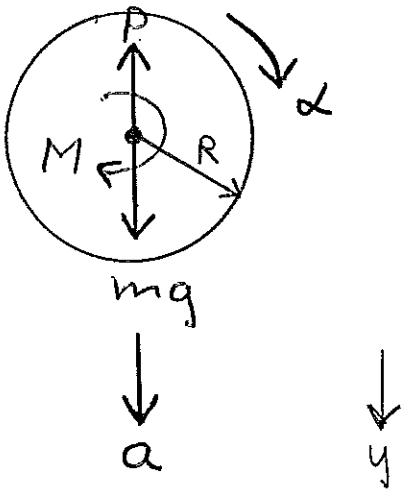
$P = mg - \frac{18}{19} mg$

$P = \frac{1}{19} mg$

Sual: $\alpha = \frac{6g}{19R}$ $\qquad P = \frac{1}{19} mg$

3/





$$\Sigma M_0 = I_0 \cdot \alpha$$

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$$\alpha = \frac{6g}{19R}$$

Skivan

$$\Sigma F_y = m a_y$$

$$mg - P = ma \qquad a = 3R\alpha = 3R \cdot \frac{6g}{19R} = \frac{18}{19} g$$

$$mg - P = m \cdot \frac{18}{19} g$$

$$P = mg - \frac{18}{19} mg$$

$$P = \frac{1}{19} mg$$

Svar: $\alpha = \frac{6g}{19R}$ $\quad P = \frac{1}{19} mg$

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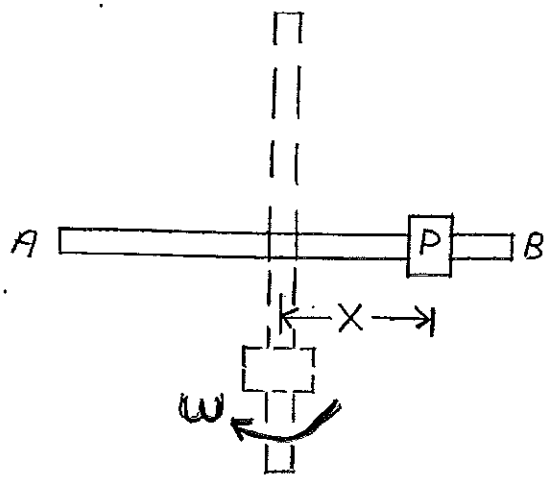
4/



$m_{AB} = m_P = 4$ kg
$l = 1{,}0$ m
a) Bestäm $w = w(x$
b) Bestäm X så att w får max

a/

Energiekvationen

$u = \Delta T + \Delta V_g + \Delta V_e$

$\Delta T = \frac{1}{2} I_0 w^2 - 0 = \frac{1}{2} w^2 \left(\frac{1}{12} m_{AB} l^2 + m_P X^2 \right) =$

$= \frac{1}{2} w^2 \left(\frac{1}{12} \cdot 4 \cdot 1^2 + 4X^2 \right) = w^2 \left(\frac{1}{6} + 2X^2 \right)$

$\Delta V_g = - m_P g X = - 4gX$

$0 = w^2 \left(\frac{1}{6} + 2X^2 \right) - 4gX = 0$

$w^2 = \frac{4gX}{\frac{1}{6} + 2X^2} = \frac{24gX}{1 + 12X^2}$

$\underline{w = \sqrt{\frac{24gX}{1+12X^2}}}$

$\frac{dw}{dX} = \frac{1}{2} \left(\frac{24gX}{1+12X^2} \right)^{-\frac{1}{2}} \cdot \frac{24g(1+12X^2 - 24gX \cdot 24X}{(1+12X^2)^2} =$

$= \frac{1}{2\sqrt{\frac{24gX}{1+12X^2}}} \cdot \frac{24g(1-12X^2)}{(1+12X^2)^2}$

$\frac{dw}{dX} = 0$ då $1 - 12X^2 = 0 \Rightarrow X = \pm \frac{1}{\sqrt{12}} \approx \underline{0{,}289 \text{ m}}$

Svar: a) $w = \sqrt{\frac{24gX}{1+12X^2}}$ b) w_{max} då $X = 0{,}289$

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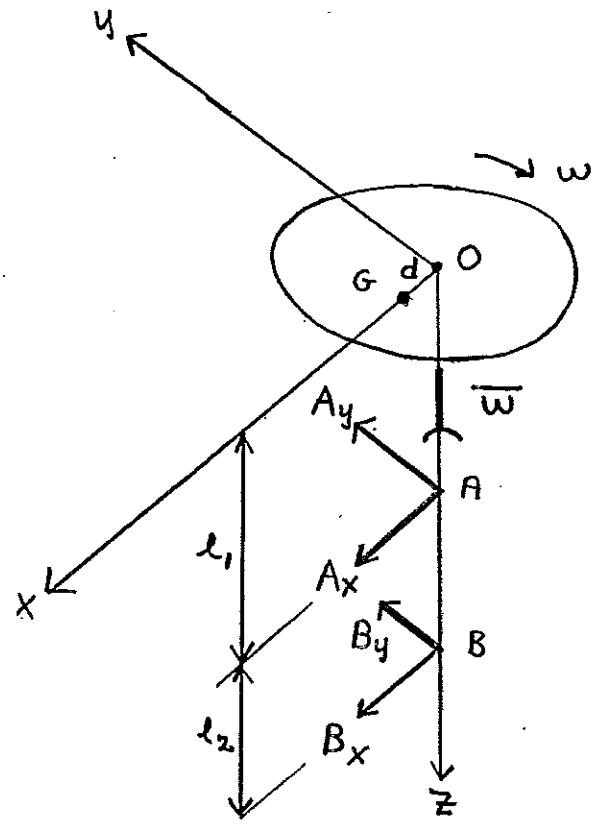
5/



Givet: $m = 6$ kg
$d = 5 \cdot 10^{-5}$ m
$l_1 = 0{,}15$ m
$l_2 = 0{,}2$ m
$\omega = \frac{2\pi N}{60} =$
$= 1047$ rad/s

Sökes: Reaktionskrafterna vid A och B p.g.a. den dynamiska obalansen.

$\overline{w}$ uttryckt i xyz

$$\Sigma \overline{M}_0 = \left(\frac{d\overline{H}_0}{dt}\right)_{XYZ} = \left(\frac{d\overline{H}_0}{dt}\right)_{xyz} + \overline{\omega} \times \overline{H}_0 \qquad \overline{\omega} = \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix}$$

$$\overline{H}_0 = \begin{Bmatrix} - & - & -I_{xz} \\ - & - & -I_{yz} \\ - & - & I_{zz} \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} = \begin{Bmatrix} -I_{xz}\,\omega \\ -I_{yz}\,\omega \\ I_{zz}\,\omega \end{Bmatrix}$$

$I_{xz} = I_{yz} = 0$ symmetri

I_{zz}

$$\overline{H}_0 = \begin{Bmatrix} 0 \\ 0 \\ I_{zz}\,\omega \end{Bmatrix}$$

$$\Sigma \overline{M}_0 = \frac{d}{dt} \begin{Bmatrix} 0 \\ 0 \\ I_{zz}\,\omega \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \begin{Bmatrix} 0 \\ 0 \\ I_{zz}\,\omega \end{Bmatrix}$$

$$\Sigma \overline{M}_0 = \overline{0} + \overline{0}$$

forts

5 forts.

$$\begin{Bmatrix} -A_y l_1 - B_y (l_1 + l_2) \\ A_x l_1 + B_x (l_1 + l_2) \\ 0 \end{Bmatrix} = \overline{0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

① $-A_y l_1 - B_y (l_1 + l_2) = 0$

② $A_x l_1 + B_x (l_1 + l_2) = 0$

$$\Sigma \overline{F} = m \overline{a}_G \qquad \overline{a}_G = \begin{Bmatrix} -d\omega^2 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} A_x + B_x \\ A_y + B_y \\ 0 \end{Bmatrix} = m \begin{Bmatrix} -d\omega^2 \\ 0 \\ 0 \end{Bmatrix}$$

③ $A_x + B_x = -m d\omega^2$

④ $A_y + B_y = 0$

① $-0{,}15 A_y - 0{,}35 B_y = 0$

② $0{,}15 A_x + 0{,}35 B_x = 0$

③ $A_x + B_x = -328{,}9$

④ $A_y = -B_y$

④ och ① ger $B_y = 0$

④ $A_y = 0$

② och ③ ger $A_x = -575{,}6\ N$ $\quad B_x = 246{,}7\ N$

Svar: $A = 576\ N \quad B = 247\ N$