# LÖSNINGAR

# TILL

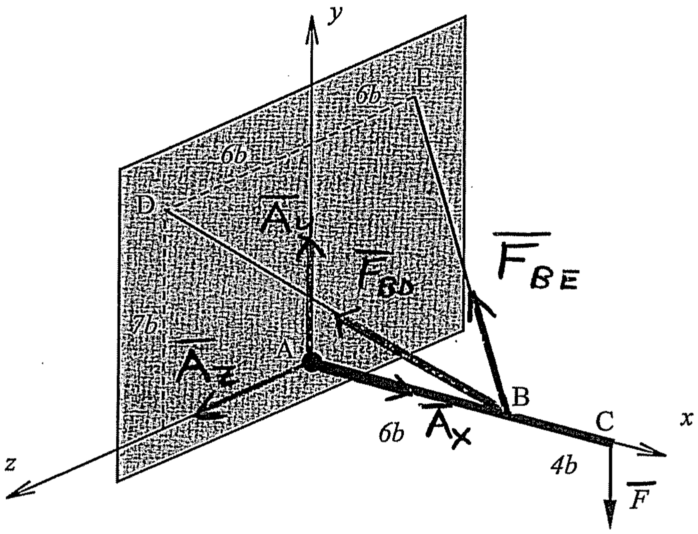
# TENTAMEN I

# MEKANIK TMMI39 f.k.

# 091219

1)

09/2/19




$$\Sigma \overline{M}_A = \overline{0}$$

$$\Sigma \overline{M}_A = \overline{r}_{AB} \times \overline{F}_{BD} + \overline{r}_{AB} \times \overline{F}_{BE} + \overline{r}_{AC} \times \overline{F}$$

$$\overline{r}_{AB} = 6b\,\overline{i} \qquad \overline{r}_{AC} = 10b\,\overline{i}$$

$$\overline{F}_{BD} = F_{BD} \cdot \frac{\overline{BD}}{|BD|} = F_{BD} \cdot \frac{-6b\,\overline{i} + 7b\,\overline{j} + 6b\,\overline{k}}{\sqrt{(6b)^2 + (7b)^2 + (6b)^2}} =$$

$$= \frac{F_{BD}}{11} (-6\,\overline{i} + 7\,\overline{j} + 6\,\overline{k})$$

$$\overline{F}_{BE} = \frac{F_{BE}}{11} (-6\,\overline{i} + 7\,\overline{j} - 6\,\overline{k})$$

$$\overline{F} = -F\,\overline{j} = -1\,\overline{j} = -\overline{j}\ kN$$

$$\Sigma \overline{M}_A = \frac{F_{BD}}{11} \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 6b & 0 & 0 \\ -6 & 7 & 6 \end{vmatrix} + \frac{F_{BE}}{11} \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 6b & 0 & 0 \\ -6 & 7 & -6 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 10b & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \overline{0}$$

$$\begin{Bmatrix} 0 \\ -\frac{F_{BD}}{11} \cdot 36b \\ \frac{F_{BD}}{11} \cdot 42b \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{F_{BE}}{11} \cdot 36b \\ \frac{F_{BE}}{11} \cdot 42b \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -10b \end{Bmatrix} = \overline{0}$$

y-led ① $\quad -\frac{F_{BD}}{11} \cdot 36b + \frac{F_{BE}}{11} \cdot 36b = 0 \Rightarrow F_{BD} = F_{BE}$

z-led ② $\quad \frac{F_{BD}}{11} \cdot 42b + \frac{F_{BE}}{11} \cdot 42b - 10b = 0$

② $\quad F_{BD} = \frac{55}{42} = 1{,}31\ kN$

Svar: $F_{BD} = F_{BE} = 1{,}31\ kN$

2)

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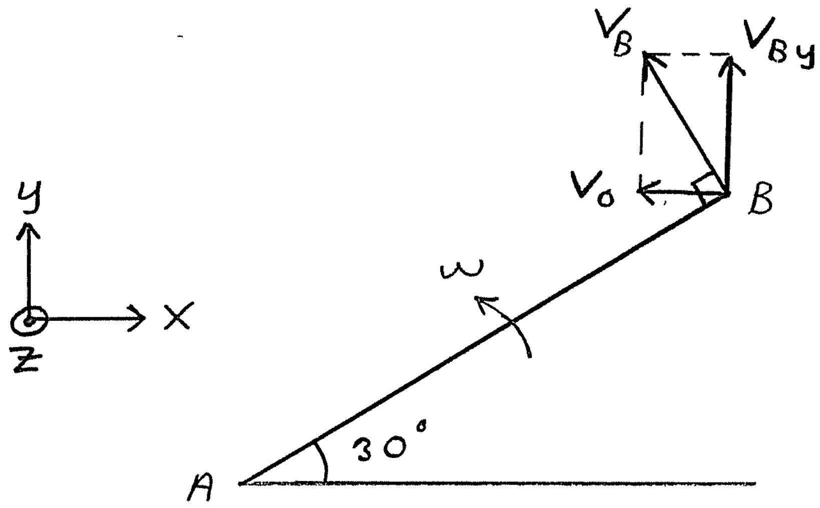


$L = 0,5\ m$

$V_0 = 2\ m/s$

$$\overline{V_B} = \overline{\omega} \times \overline{AB}$$

$$\overline{\omega} = \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \qquad \overline{AB} = \begin{Bmatrix} 0,5 \cos 30° \\ 0,5 \sin 30° \\ 0 \end{Bmatrix}$$

$$\overline{V_B} = \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix} \times \begin{Bmatrix} 0,5 \cos 30° \\ 0,25 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0,25\,\omega \\ 0,5 \cos 30° \cdot \omega \\ 0 \end{Bmatrix}$$

$$\begin{cases} V_{Bx} = -V_0 = -2\ m/s \\ V_{Bx} = -0,25\,\omega \end{cases}$$

$$-2 = -0,25\,\omega$$

$$\omega = 8\ rad/s$$

**Svar: $\omega = 8\ rad/s$**

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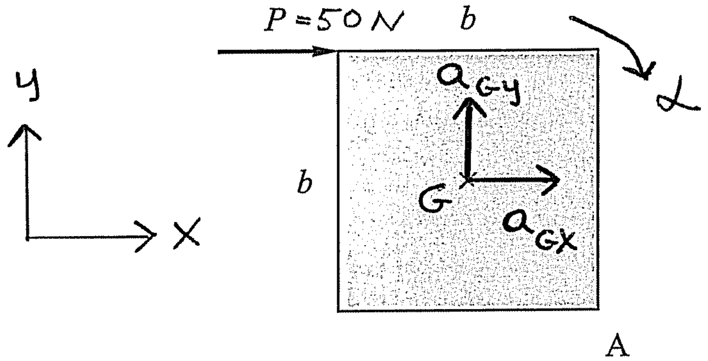


$\omega = 0$

$b = 1{,}0$ m

$m = 5$ kg

$$\bar{a}_A = \bar{a}_G + \bar{\alpha} \times \overline{GA} + \bar{\omega} \times (\bar{\omega} \times \overline{GA})$$

$$\bar{a}_A = \bar{a}_G + \bar{\alpha} \times \overline{GA}$$

$\sum F_x = m\, a_{Gx}$ ① $50 = 5\, a_{Gx}$

$\sum F_y = m\, a_{Gy}$ ② $0 = 5\, a_{Gy}$

$\sum M_G = I \cdot \alpha$ $\curvearrowright G$ ③ $50 \cdot 0{,}5 = I_G \cdot \alpha$

$$I_G = \frac{1}{12} m (a^2 + b^2) =$$

$$= \frac{1}{12} \cdot 5 (1^2 + 1^2) = \frac{5}{6}$$

$$= 0{,}8333 \text{ kgm}^2$$

① $a_{Gx} = 10$ m/s²

② $a_{Gy} = 0$

③ $\alpha = 30$ rad/s²

$$\bar{a}_A = \begin{Bmatrix} 10 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -30 \end{Bmatrix} \times \begin{Bmatrix} 0{,}5 \\ -0{,}5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -15 \\ -15 \\ 0 \end{Bmatrix}$$

$$\bar{a}_A = \begin{Bmatrix} -5 \\ -15 \\ 0 \end{Bmatrix} \qquad |\bar{a}_A| = \sqrt{(-5)^2 + (-15)^2} =$$

$$= \sqrt{250} \approx 15{,}8 \text{ m/s}^2$$

Svar: $a_A = 15{,}8$ m/s²

4)

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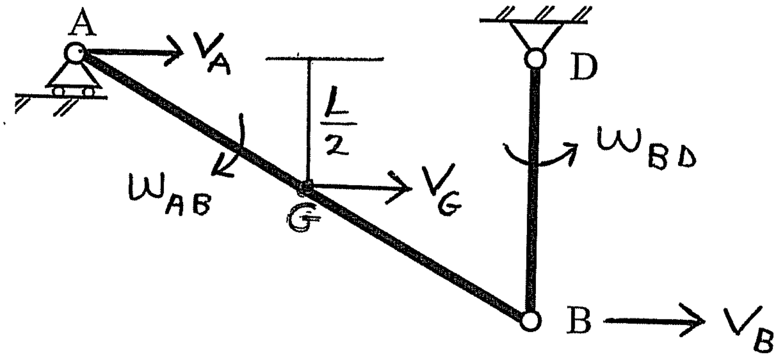


Energiekvationen

$U = \Delta T + \Delta V_g + \Delta V_e$

$U = \Delta V_e = 0$

<u>AB</u>

$V_A$ parallell med $V_B \Rightarrow \omega_{AB} = 0$

$V_G = V_A$

$\Delta T = T_2 - T_1 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2 - 0 =$

$= \frac{1}{2} \cdot 2 m V_A^2 + 0 = \underline{m V_A^2}$

$\Delta V_g = -2mg \cdot \frac{L}{2} = \underline{-mgL}$

<u>BD</u>

$V_B = V_A = \omega_{BD} \cdot L \Rightarrow \omega_{BD} = \frac{V_A}{L}$

$\Delta T = \frac{1}{2} I_0 \omega^2 = \frac{1}{2} \cdot \frac{1}{3} m L^2 \cdot \left(\frac{V_A}{L}\right)^2 = \underline{\frac{1}{6} m V_A^2}$

$\Delta V_g = -mg \cdot \frac{L}{2} = \underline{-\frac{1}{2} mgL}$

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$$U = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = \cancel{m} v_A^2 - \cancel{m} g L + \frac{1}{6} \cancel{m} v_A^2 - \frac{1}{2} \cancel{m} g L$$

$$\frac{7}{6} v_A^2 = \frac{3}{2} g L$$

$$v_A^2 = \frac{9}{7} g L$$

$$v_A = 3 \sqrt{\frac{gL}{7}}$$

Svar: $v_A = 3\sqrt{\frac{gL}{7}}$

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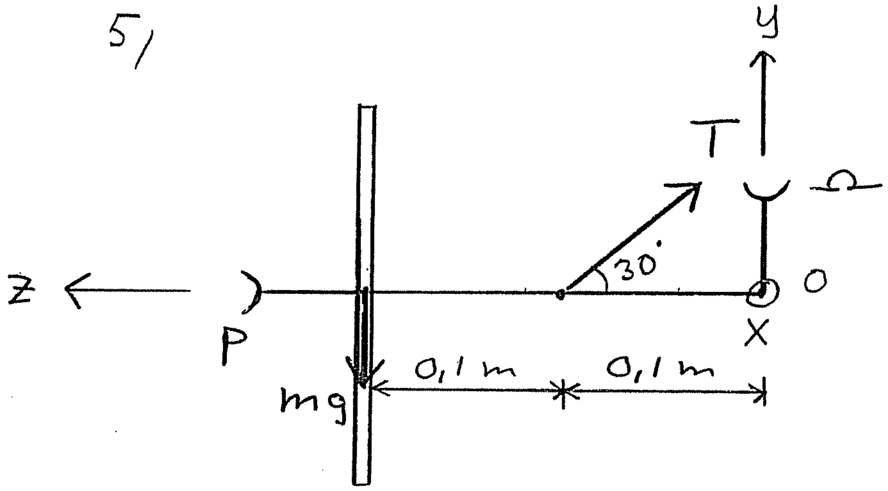


I punkten O finns $O_x$, $O_y$ och $O_z$.

$p = 100$ rad/s
$\Omega = 0{,}5$ rad/s
$r = b = 0{,}1$ m
$m = 1{,}5$ kg

$\bar{\Omega}$ vrider $xyz$

$$\bar{\Omega} = \begin{Bmatrix} 0 \\ 0{,}5 \\ 0 \end{Bmatrix}$$

$$\bar{\omega} = \bar{\Omega} + \bar{p} = \begin{Bmatrix} 0 \\ 0{,}5 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 100 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0{,}5 \\ 100 \end{Bmatrix}$$

$$\bar{M}_0 = \left(\frac{d\bar{H}_0}{dt}\right)_{xyz} + \bar{\Omega} \times \bar{H}_0$$

$$\bar{H}_0 = \bar{I}_0 \bar{\omega} = \begin{Bmatrix} - & 0 & 0 \\ - & I_{yy} & 0 \\ - & \underset{\text{symmetri}}{0} & I_{zz} \end{Bmatrix} \begin{Bmatrix} 0 \\ 0{,}5 \\ 100 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0{,}5\, I_{yy} \\ 100\, I_{zz} \end{Bmatrix}$$

$$\begin{Bmatrix} 1{,}5g \cdot 0{,}2 - T\sin 30^\circ \cdot 0{,}1 \\ 0 \\ 0 \end{Bmatrix} = \bar{0} + \begin{Bmatrix} 0 \\ 0{,}5 \\ 0 \end{Bmatrix} \times \begin{Bmatrix} 0 \\ 0{,}5\, I_{yy} \\ 100\, I_{zz} \end{Bmatrix}$$

$$\begin{Bmatrix} 0{,}3g - 0{,}05T \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 50\, I_{zz} \\ 0 \\ 0 \end{Bmatrix}$$

$$I_{zz} = \frac{1}{2} m r^2 = \frac{1}{2} \cdot 1{,}5 \cdot 0{,}1^2 = 0{,}0075 \text{ kgm}^2$$

$$0{,}3g - 0{,}05T = 50 \cdot 0{,}0075$$

$$T = \frac{0{,}3g - 0{,}375}{0{,}05} = 51{,}36\,N$$

<u>Svar: T = 51,4 N</u>