LÖSNINGAR

TILL

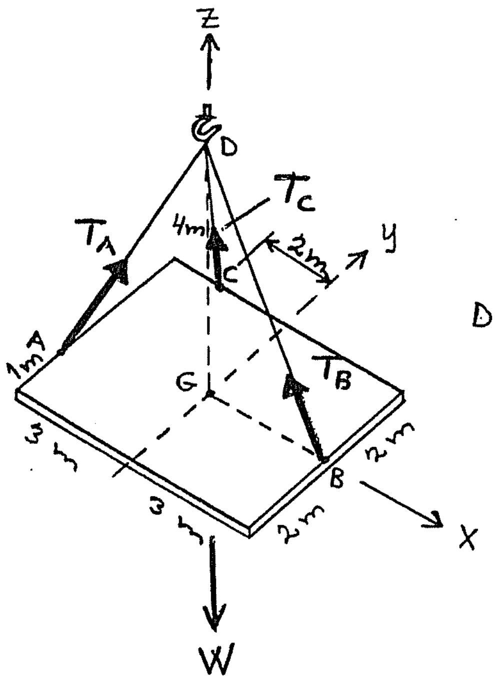
TENTAMEN I

MEKANIK TMMI39 f.k.

100408

100408

1)



DG = 4 m

Antag att vi har största kraften i BD,
d.v.s. $T_B = 20$ kN

$$\Sigma \overline{F} = \overline{0}$$

$$\overline{T_A} + \overline{T_B} + \overline{T_C} + \overline{W} = \overline{0}$$

$$\overline{T_A} = T_A \overline{n_{AD}} = T_A \frac{\overline{AD}}{AD} = T_A \frac{3\overline{i} + \overline{j} + 4\overline{k}}{\sqrt{26}}$$

$$\overline{T_B} = T_B \overline{n_{BD}} = T_B \frac{\overline{BD}}{BD} = 20 \frac{-3\overline{i} + 4\overline{k}}{5}$$

$$\overline{T_C} = T_C \overline{n_{CD}} = T_C \frac{\overline{CD}}{CD} = T_C \frac{2\overline{i} - 2\overline{j} + 4\overline{k}}{\sqrt{24}}$$

$$\overline{W} = -W\overline{k}$$

x-led ① $T_A \cdot \frac{3}{\sqrt{26}} + 20 \cdot \left(-\frac{3}{5}\right) + T_C \cdot \frac{2}{\sqrt{24}} = 0$

y-led ② $T_A \cdot \frac{1}{\sqrt{26}} + T_C \cdot \left(-\frac{2}{\sqrt{24}}\right) = 0 \Rightarrow \frac{1}{\sqrt{26}} T_A = \frac{2}{\sqrt{24}} T_C$ insättes i ①

z-led ③ $T_A \cdot \frac{4}{\sqrt{26}} + 20 \cdot \frac{4}{5} + T_C \cdot \frac{4}{\sqrt{24}} - W = 0$

① $\frac{6}{\sqrt{24}} T_C + \frac{2}{\sqrt{24}} T_C = 12 \Rightarrow \underline{T_C = 3\sqrt{6}} \Rightarrow \underline{T_A = 3\sqrt{26}}$

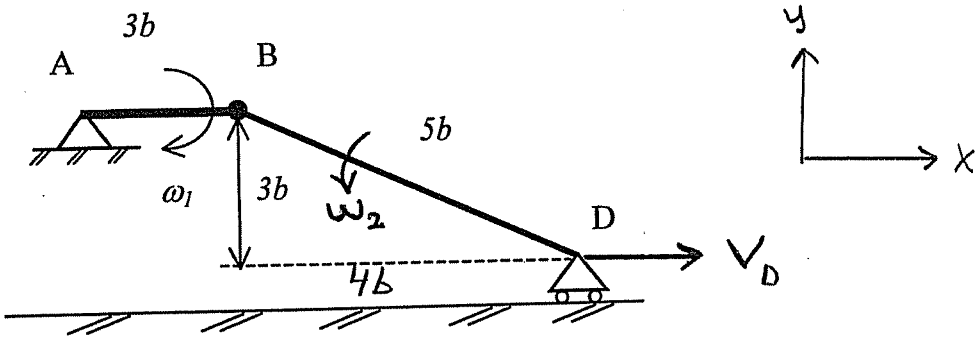
$T_B > T_A > T_C$; antagandet korrekt

③ $3\sqrt{26} \cdot \frac{4}{\sqrt{26}} + 16 + 3\sqrt{6} \cdot \frac{4}{\sqrt{24}} = W \Rightarrow W = 34$

<u>Svar: $W_{max} = 34$ kN</u>

2)

100408



$$\overline{V_B} = \overline{V_D} + \overline{\omega_2} \times \overline{DB}$$

$$\overline{V_B} = \overline{\omega_1} \times \overline{AB}$$

$$\overline{V_B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & -4 \\ 3b & 0 & 0 \end{vmatrix} = \begin{Bmatrix} 0 \\ -12b \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ -12b \\ 0 \end{Bmatrix} = \begin{Bmatrix} V_D \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega_2 \end{Bmatrix} \times \begin{Bmatrix} -4b \\ 3b \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ -12b \\ 0 \end{Bmatrix} = \begin{Bmatrix} V_D \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -\omega_2 \cdot 3b \\ -\omega_2 4b \\ 0 \end{Bmatrix}$$

x-led ① $0 = V_D - \omega_2 \cdot 3b$

y-led ② $-12b = -\omega_2 4b$

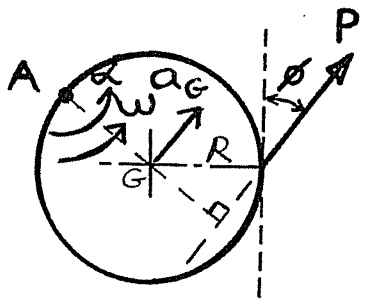
② $\omega_2 = 3$ rad/s

① $V_D = 3 \cdot 3b = 9b$

Svar: $\overline{V_D} = 9b\,\bar{i}$ m/s

3)

100408



$\omega = 0$

$\sum F = m a_G$ ① $P = m a_G$

$\sum M_G = I_G \cdot \alpha$ ② $P \cos\phi \cdot R = \frac{1}{2} m R^2 \cdot \alpha$

$\bar{a}_G = \bar{a}_A + \bar{\alpha} \times \overline{AG}$

$\bar{a}_G = \bar{\alpha} \times \overline{AG} \Rightarrow$

③ $a_G = R \cdot \alpha$

① $P = m R \alpha$

Insätter i ②

② $m R \alpha \cos\phi \cdot R = \frac{1}{2} m R^2 \alpha$

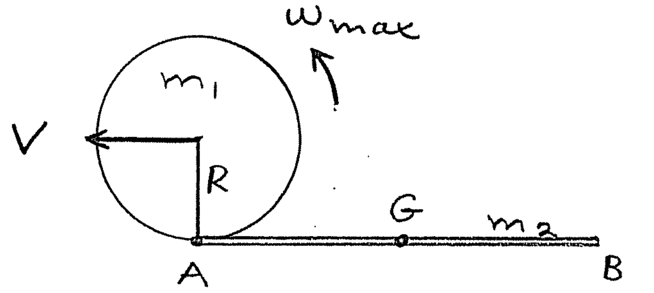
$\cos\phi = \frac{1}{2}$

$\phi = 60°$

Svar: $\phi = 60°$

4)

ω_{max} då punkten A befinner sig längst ned, dvs då stången är horisontell.



$V_{AB} = 0 \qquad \omega_{AB} = 0$

$$U = \Delta T + \Delta V_g + \Delta V_e$$

<u>Stången</u>

$\Delta T = 0$

$\Delta V_g = -m_2 g R$

<u>Skivan</u>

$$\Delta T = \frac{1}{2} m_1 v^2 + \frac{1}{2} I_G \omega_{max}^2 =$$

$$= \frac{1}{2} m_1 (R\omega_{max})^2 + \frac{1}{2} \cdot \frac{1}{2} m_1 R^2 \cdot \omega_{max}^2 =$$

$$= \frac{3}{4} m_1 R^2 \omega_{max}^2$$

$\Delta V_g = 0$

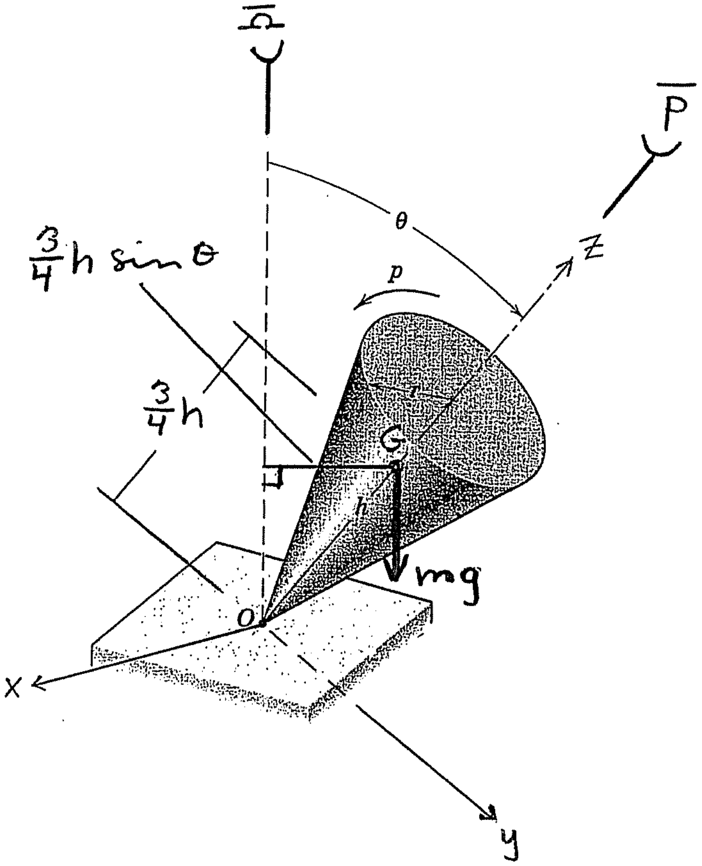
$$0 = -m_2 g R + \frac{3}{4} m_1 R^2 \omega_{max}^2$$

$$\omega_{max} = \sqrt{\frac{4 m_2 g}{3 m_1 R}}$$

Svar: $\omega_{max} = \sqrt{\frac{4 m_2 g}{3 m_1 R}}$

5)

100408



<u>Givet:</u>

$r = 0{,}12$ m

$h = 0{,}36$ m

$p = 314{,}2$ rad/s

<u>Sökes:</u>

Periodtiden τ för precessionen dvs $\tau = \frac{2\pi}{\Omega}$

$\bar{\Omega}$ vrider xyz

$$\bar{\Omega} = \begin{Bmatrix} 0 \\ -\Omega \sin\theta \\ \Omega\cos\theta \end{Bmatrix} \qquad \bar{P} = \begin{Bmatrix} 0 \\ 0 \\ p \end{Bmatrix}$$

$$\bar{\omega} = \bar{\Omega} + \bar{P} = \begin{Bmatrix} 0 \\ -\Omega\sin\theta \\ \Omega\cos\theta + p \end{Bmatrix}$$

$$\bar{M}_o = \left(\frac{dH_o}{dt}\right)_{xyz} + \bar{\Omega} \times \bar{H}_o$$

$$\bar{H}_o = \bar{I}_o\,\bar{\omega} = \begin{Bmatrix} - & 0 & 0 \\ - & I_{yy} & 0 \\ - & \underset{\text{symmetri}}{0} & I_{zz} \end{Bmatrix} \begin{Bmatrix} 0 \\ -\Omega\sin\theta \\ \Omega\cos\theta + p \end{Bmatrix}$$

$$\bar{H}_o = \begin{Bmatrix} 0 \\ -I_{yy}\Omega\sin\theta \\ I_{zz}\Omega\cos\theta + I_{zz}p \end{Bmatrix}$$

forts.

5 forts,

Momentet

$$\begin{Bmatrix} -mg\cdot\frac{3}{4}h\sin\theta \\ 0 \\ 0 \end{Bmatrix} = \bar{0} + \begin{Bmatrix} 0 \\ -\Omega\sin\theta \\ \Omega\cos\theta \end{Bmatrix} \times \begin{Bmatrix} 0 \\ -I_{yy}\Omega\sin\theta \\ I_{zz}\Omega\cos\theta + I_{zz}p \end{Bmatrix}$$

$$\begin{Bmatrix} -mg\cdot\frac{3}{4}h\sin\theta \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} (I_{yy}-I_{zz})\Omega^2\sin\theta\cos\theta - I_{zz}\Omega p\sin\theta \\ 0 \\ 0 \end{Bmatrix}$$

$$-mg\cdot\frac{3}{4}h\cancel{\sin\theta} = \cancel{(I_{yy}-I_{zz})\Omega^2\sin\theta\cos\theta} - I_{zz}\Omega p\cancel{\sin\theta}$$

$p >> \Omega$

$$mg\cdot\frac{3}{4}h = I_{zz}\Omega p$$

$$\Omega = \frac{3mg\cdot h}{4I_{zz}p} \qquad I_{zz} = \frac{3}{10}mr^2$$

$$\Omega = \frac{\cancel{3}\cancel{m}gh}{4\cdot\frac{\cancel{3}}{10}\cancel{m}r^2\cdot p}$$

$$\Omega = \frac{10gh}{4r^2p} = \frac{10g\cdot 0{,}36}{4\cdot 0{,}12^2\cdot 314{,}2} = 1{,}9514 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\Omega} = 3{,}22\text{ s}$$

$$\underline{\underline{\text{Svar: } \tau = 3{,}22\text{ s}}}$$