LÖSNINGAR

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TENTAMEN I

MEKANIK TMMI39 f.k.

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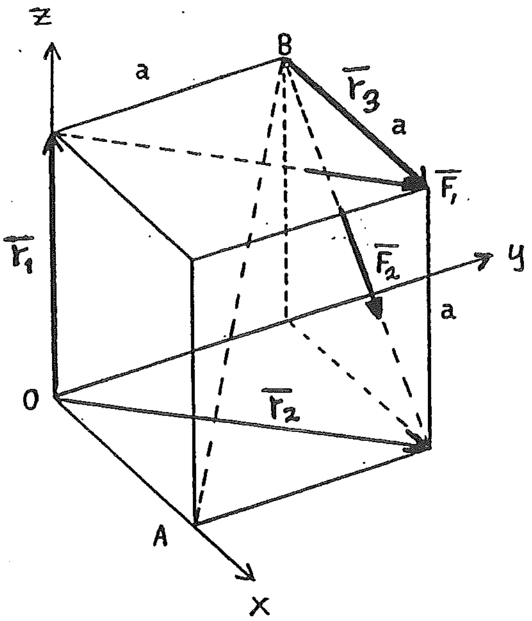
1. 100826



a)

$$F_1 = \frac{F}{\sqrt{2}}(1,1,0) \qquad F_2 = \frac{F}{\sqrt{2}}(1,0,-1)$$

$$\Sigma F = \frac{F}{\sqrt{2}}\left[(1,1,0)+(1,0,-1)\right] = \underline{\frac{F}{\sqrt{2}}(2,1,-1)}$$

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = (0,0,a) \times \frac{F}{\sqrt{2}}(1,1,0) = \frac{Fa}{\sqrt{2}}(-1,1,0)$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = (a,a,0) \times \frac{F}{\sqrt{2}}(1,0,-1) = \frac{Fa}{\sqrt{2}}(-1,1,-1)$$

$$\Sigma \bar{M} = \bar{M}_1 + M_2 = \frac{Fa}{\sqrt{2}}\left[(-1,1,0)+(-1,1,-1)\right] = \underline{\frac{Fa}{\sqrt{2}}(-2,2,-1)}$$

b)

$$M_{AB} = \bar{M}_B \circ \bar{n}_{AB}$$

$$\bar{M}_B = \bar{r}_3 \times \bar{F}_1 = (a,0,0) \times \frac{F}{\sqrt{2}}(1,1,0) = \frac{Fa}{\sqrt{2}}(0,0,1)$$

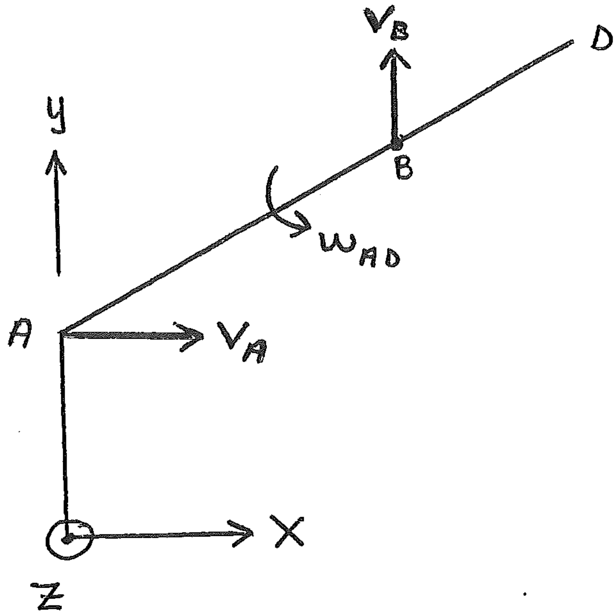
$$\bar{n}_{AB} = \frac{1}{\sqrt{3}}(-1,1,1)$$

$$M_{AB} = \frac{Fa}{\sqrt{2}}(0,0,1) \circ \frac{1}{\sqrt{3}}(-1,1,1) = \frac{Fa}{\sqrt{6}}$$

Svar: a) $\Sigma \bar{F} = \frac{F}{\sqrt{2}}(2,1,-1) \qquad \Sigma \bar{M} = \frac{Fa}{\sqrt{2}}(-2,2,-1)$

b) $M_{AB} = \frac{Fa}{\sqrt{6}}$

2)



① $\overline{V_D} = \overline{V_B} + \overline{V_{D/B}} = \overline{V_B} + \overline{\omega_{AD}} \times \overline{BD}$ Bestäm $\overline{\omega_{AD}}$

② $\overline{V_B} = \overline{V_A} + \overline{V_{B/A}} = \overline{V_A} + \overline{\omega_{AD}} \times \overline{AB} \Rightarrow \overline{\omega_{AD}}$

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② $\overline{V_B} = 40\overline{j} \quad \overline{V_A} = V_A\overline{i} \quad \overline{\omega_{AD}} = \omega_{AD}\overline{k}$

$\overline{AB} = 160\cos 30^\circ \overline{i} + 160 \sin 30^\circ \overline{j} = 80\sqrt{3}\,\overline{i} + 80\overline{j}$

$$\overline{\omega_{AD}} \times \overline{AB} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 0 & \omega_{AD} \\ 80\sqrt{3} & 80 & 0 \end{vmatrix} = -80\,\omega_{AD}\overline{i} + 80\sqrt{3}\,\omega_{AD}\overline{j}$$

② $$\begin{Bmatrix} 0 \\ 40 \\ 0 \end{Bmatrix} = \begin{Bmatrix} V_A \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -80\,\omega_{AD} \\ 80\sqrt{3}\,\omega_{AD} \\ 0 \end{Bmatrix}$$ $\omega_{AD} = 0{,}2887$ rad/s

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① $\overline{BD} = 80\cos 30^\circ \overline{i} + 80 \sin 30^\circ \overline{j} = 40\sqrt{3}\,\overline{i} + 40\overline{j}$

$$\overline{\omega_{AD}} \times \overline{BD} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 0 & 0{,}2887 \\ 40\sqrt{3} & 40 & 0 \end{vmatrix} = -11{,}548\overline{i} + 20\overline{j}$$

① $$\overline{V_D} = \begin{Bmatrix} 0 \\ 40 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -11{,}548 \\ 20 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -11{,}548 \\ 60 \\ 0 \end{Bmatrix}$$

$V_D = |\overline{V_D}| = \sqrt{11{,}548^2 + 60^2} = 61{,}1$ mm/s

<u>Svar: $V_D = 61{,}1$ mm/s</u>

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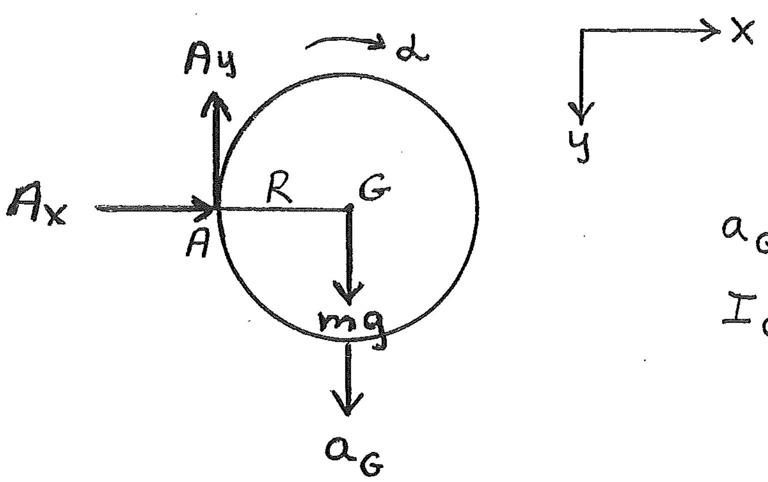
3)



$a_G = R\alpha$

$I_G = \frac{1}{2} m R^2$

$\Sigma F_x = m a_x = 0$

$\Sigma F_y = m a_G$

$\Sigma M_A = I_G \alpha + m a_G d_\perp \quad \overset{\curvearrowright}{A}$

① $A_x = 0$

② $mg - A_y = mR\alpha$

③ $mgR = \frac{1}{2} m R^2 \alpha + mR\alpha \cdot \underset{d_\perp}{R}$

① $\underline{A_x = 0}$

② $A_y = mg - mR\alpha$

③ $g = \frac{1}{2} R\alpha + R\alpha$

$g = \frac{3}{2} R\alpha$

$\alpha = \frac{2g}{3R}$ insättes i ②

② $A_y = mg - mR \cdot \frac{2g}{3R}$

$A_y = mg - \frac{2}{3} mg$

$A_y = \frac{1}{3} mg$

$\underline{A_y} = \frac{1}{3} \cdot 3g = g = \underline{9{,}81\ N}$

Svar: $A_x = 0$, $A_y = 9{,}81\ N$

4)

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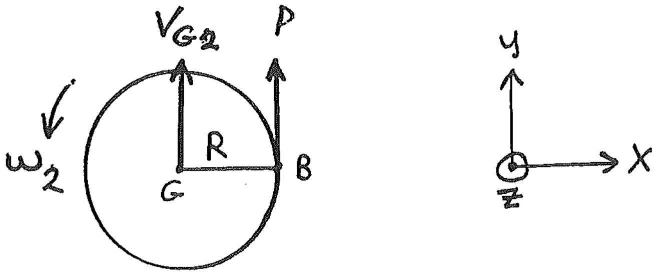

<u>Sökes:</u> $V_B = V_B(t)$

<u>Givet:</u> m, R, P

$$\int \Sigma \bar{F}\, dt = \bar{G}_2 - \bar{G}_1 \qquad \uparrow ① \int_0^t P\, dt = m V_{G2} - m \cdot \overset{0}{V_{G1}}$$

$$\int \Sigma \bar{M}_G\, dt = \bar{H}_{G2} - \bar{H}_{G1} \qquad \circlearrowleft G\ ② \int_0^t P \cdot R\, dt = I_G \omega_2 - I_G \overset{0}{\omega_1}$$

$$① \quad Pt = m V_{G2}$$

$$② \quad PRt = \frac{1}{2} m R^2 \cdot \omega_2$$

$$① \quad V_{G2} = \frac{Pt}{m}$$

$$② \quad \omega_2 = \frac{2Pt}{mR}$$

$$\bar{V}_B = \bar{V}_G + \bar{V}_{B/G} = \bar{V}_G + \bar{\omega} \times \overline{GB}$$

$$\bar{V}_B = \begin{Bmatrix} 0 \\ \frac{Pt}{m} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \frac{2Pt}{mR} \end{Bmatrix} \times \begin{Bmatrix} R \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{3Pt}{m} \\ 0 \end{Bmatrix}$$

$$\bar{V}_B = \frac{3Pt}{m} \bar{j}$$

<u>Svar: $V_B = \frac{3Pt}{m}$</u>

5)

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Givet: $m = 0,25$ kg $\quad R = 0,060$ m

$\omega_1 = 200$ rad/s $\quad \omega_2 = 60$ rad/s

Sökes: $\overline{M}_O = \overline{M}_G$

$\overline{\Omega} = \overline{\omega}_2$ under xyz

$$\overline{M}_O = \overline{M}_G = \left(\frac{d\overline{H}_G}{dt}\right)_{XYZ} = \left(\frac{d\overline{H}_G}{dt}\right)_{xyz} + \overline{\Omega} \times \overline{H}_G$$

$$\overline{\Omega} = \overline{\omega}_2 = \begin{Bmatrix} 0 \\ 0 \\ \omega_2 \end{Bmatrix}$$

$$\overline{\omega} = \overline{\omega}_1 + \overline{\omega}_2 = \begin{Bmatrix} -\omega_1 \\ 0 \\ \omega_2 \end{Bmatrix}$$

$$\overline{H}_G = \overline{I}_G \, \overline{\omega} = \begin{bmatrix} I_{xx} & - & 0 \\ 0 & - & 0 \\ 0 & - & I_{zz} \end{bmatrix} \begin{bmatrix} -\omega_1 \\ 0 \\ \omega_2 \end{bmatrix} = \begin{Bmatrix} -\omega_1 I_{xx} \\ 0 \\ \omega_2 I_{zz} \end{Bmatrix}$$

$$\overline{\Omega} \times \overline{H}_G = \begin{Bmatrix} 0 \\ 0 \\ \omega_2 \end{Bmatrix} \times \begin{Bmatrix} -\omega_1 I_{xx} \\ 0 \\ \omega_2 I_{zz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\omega_1 \omega_2 I_{xx} \\ 0 \end{Bmatrix}$$

$I_{xx} = mR^2$ (ring)

$$\overline{M}_O = \overline{M}_G = \overline{0} + \begin{Bmatrix} 0 \\ -\omega_1 \omega_2 m R^2 \\ 0 \end{Bmatrix}$$

$$\overline{M}_O = -\omega_1 \omega_2 m R^2 \overline{j} = -200 \cdot 60 \cdot 0,25 \cdot 0,060^2 \overline{j} \text{ Nm}$$

$$\overline{M}_O = -10,8 \overline{j} \text{ Nm}$$

Svar: $\overline{M}_O = -10,8 \overline{j}$ Nm